

Active-to-absorbing phase transition subjected to the velocity fluctuations in the frozen limit case

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Abstract

The directed bond percolation process is studied in the presence of compressible velocity fluctuations with long-range correlations. We discuss a construction of a field theoretic action and a way of obtaining its large scale properties using the perturbative renormalization group. The most interesting results for the frozen velocity limit are given.

Аннотация

Исследуется процесс прямой перколяции в присутствии сжимаемых флуктуаций скорости с крупномасштабными корреляциями. Обсуждается построение теоретико-полевого действия и способ получения его свойств в области больших масштабов с использованием пертурбативной ренормализационной группы. Представлены наиболее интересные результаты в пределе замороженной турбулентности.

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1 Introduction

Non-equilibrium continuous phase transitions have been and still are an object of intense research activity. Underlying dynamic laws are responsible for a more diverse behavior in contrast to the equilibrium models. One of the most prominent examples is the directed bond percolation [1, 2] process that can be interpreted as a simple spreading process of an infection disease [3]. A distinguished feature is the presence of non-equilibrium phase transition between the active and absorbing state. There the correlation diverges and large scale properties become independent on the microscopic details. It has been known that phase transitions are quite sensitive to additional disturbances such as quenched disorder [4] or long-range interactions [5]. From practical point of view this might be a reason why there are not so many experimental realizations for the percolation process [6].

In this paper we assume that the effect of environment can be simulated by advective velocity fluctuations with finite correlation in time and compressibility taken into account. Following the works [7, 8] we consider a generalization of the original Kraichnan model where the advection-diffusion problem of non-interacting admixture has been studied. As some progress in this direction has already been made [9, 10, 11, 12, 13, 14], our main aim here is to elucidate in detail the case of frozen velocity limit [8, 15]. The main theoretical tool is the field-theoretic approach [16] with subsequent Feynman diagrammatic technique and renormalization group (RG) approach, which allows us to determine the large-scale behavior, which is of special interest from macroscopic point of view.

The paper is organized as follows. In Sec. 2, we introduce a field-theoretic version of the problem and in Sec. 3 we present an analysis of possible regimes, which corresponds to the fixed points (FP) of the RG equations.

2 Field-theoretic model

The large scale properties of the directed bond percolation process [1, 2] are captured by the following Langevin equation

$$\partial_t \psi = D_0(\nabla^2 - \tau_0)\psi - \lambda_0 D_0 \psi^2/2 + \eta, \quad (1)$$

where the field $\psi(t, \mathbf{x})$ corresponds to the density of percolating agents, $\partial_t = \partial/\partial t$ is the time derivative, ∇^2 is the Laplace operator, D_0 is the diffusion constant, λ_0 is the coupling constant and τ_0 controls a deviation from the

criticality. The noise field $\eta(t, \mathbf{x})$ stems from the coarse graining procedure and mimics the influence of fast degrees of freedom. It is assumed that η is a random Gaussian variable with zero mean and correlator

$$\langle \eta(t_1, \mathbf{x}_1) \eta(t_2, \mathbf{x}_2) \rangle = g_0 D_0 \psi(t_1, \mathbf{x}_1) \delta(t_1 - t_2) \delta^{(d)}(\mathbf{x}_1 - \mathbf{x}_2), \quad (2)$$

where $\delta^{(d)}(\mathbf{x})$ is the d -dimensional delta function. The amplitude factor $\propto \psi$ in (2) ensures that the fluctuations cease in the absorbing state $\psi = 0$.

An advection process can be introduced in the standard fashion using the replacement of time derivative ∂_t by the Lagrangian derivative $\nabla_t \equiv \partial_t + (\mathbf{v} \cdot \nabla)$. A basic idea of the Kraichnan model [7, 8] is that the velocity field \mathbf{v} is a random Gaussian variable with zero mean and a translationally invariant correlator [8] conveniently given in the Fourier representation as follows

$$\langle v_i v_j \rangle_0(\omega, \mathbf{k}) = [P_{ij}^k + \alpha Q_{ij}^k] D_v(\omega, \mathbf{k}), \quad D_v(\omega, \mathbf{k}) = \frac{g_{10} u_{10} D_0^3 k^{4-d-y-\eta}}{\omega^2 + u_{10}^2 D_0^2 (k^{2-\eta})^2}. \quad (3)$$

Here, $P_{ij}^k = \delta_{ij} - k_i k_j / k^2$ is a transverse and $Q_{ij}^k = k_i k_j / k^2$ a longitudinal projection operator. Further, the function D_v is known as a kernel, where $k = |\mathbf{k}|$ and a positive parameter $\alpha > 0$ can be interpreted as a deviation [8] from the incompressibility condition $\nabla \cdot \mathbf{v} = 0$. The coupling constant g_{10} and the exponent y describe the energy spectrum [7, 8] of the velocity fluctuations. The constant $u_{10} > 0$ and the exponent η are related to the characteristic frequency of the mode with wavelength k . The kernel function for the correlator (3) contains special cases: rapid-change model, frozen velocity ensemble and turbulence advection (see [7, 8]). Our aim here is to analyze the case of the frozen velocity limit, which is obtained in the limit $u_{10} \rightarrow 0$. It bears resemblance to a model of random walks in a random environment with long-range correlations [15]. The correlator (3) attains then the form

$$\langle v_i v_j \rangle_0(\omega, \mathbf{k}) \rightarrow [P_{ij}^k + \alpha Q_{ij}^k] g_0 D_0^2 \pi \delta(\omega) k^{2-d-y}. \quad (4)$$

In this limit velocity correlator does not depend on the time difference $(t - t')$. The name “frozen” is motivated by the detailed analysis [7], where it has been shown that in this limit the correlation time of the velocity field $t_v(k)$ as a function of momentum scale k is much larger than the correlation time of the advecting scalar quantity $t_\psi(k)$ and effectively t_v can be set to infinity. Hence from the point of view of ψ field the velocity fluctuations are not evolving in time, hence “frozen”.

The stochastic problem (1) and (3) is amenable to the full machinery of the quantum field theory methods as Feynman diagrammatic expansion and

renormalization group method. The latter can be done in a straightforward fashion following the well-established theory [16] and constitutes our main theoretical tool. According to a general theorem [16], the stochastic problem is equivalent to the field theoretic model of a doubled set of fields $\tilde{\psi}, \psi$ with Janssen-De Dominicis functional written in a compact form as

$$\begin{aligned} \mathcal{J}[\tilde{\psi}, \psi, \mathbf{v}] = & \tilde{\psi}[\nabla_t + D_0(\tau_0 - \nabla^2)]\psi + D_0\lambda_0[\psi - \tilde{\psi}]\tilde{\psi}\psi/2 + vD_v^{-1}v/2, \\ & + a_0\tilde{\psi}(\nabla \cdot \mathbf{v})\psi - \frac{u_{20}}{2D_0}\tilde{\psi}\psi\mathbf{v}^2. \end{aligned} \quad (5)$$

We use a condensed notation in which integrals over the spatial variable x and the time variable t and summation over repeated index are implicitly assumed. For example, the first term on the right hand side actually stands for $\int dt \int d^d\mathbf{x} \tilde{\psi}(t, \mathbf{x})\partial_t\psi(t, \mathbf{x})$. The last two terms in the action (5) have been added in order to ensure renormalizability of the model. Their presence can be tracked down to the broken Galilean invariance of the velocity ensemble and assumed compressibility. These directly lead to the more involved numerical analysis of the flow equations than previously considered models [9, 10, 11, 12]. It can be proven [14] that the field theoretic action (5) is closed with respect to the renormalization group analysis and no other interactions than that already involved in (5) are generated. Thus the model (5) is multiplicatively renormalizable. Then according to the general theory of the RG group one can exploit the correlation between ultraviolet and infrared (IR) singularities for a logarithmic theory and information about large-scale (macroscopic) behavior can be extracted. In contrast to the standard ε -expansion in φ^4 -theory we deal here with the three parameter expansion (ε, y, η) , which are considered to be of the same order of magnitude. Here as usual $\varepsilon = 4 - d$ is a deviation from the upper critical dimension $d_c = 4$ [2].

3 Results of the RG analysis

In the frozen velocity limit [8] the fixed point coordinate of the charge u_1^* is zero. The RG analysis reveals [14] existence of eight possible regimes, from which only three points FP_1 , FP_2 and FP_3 are IR stable and hence possibly observable on the macroscopic scale. The physical interpretation of the stable regimes is the following: the fixed point FP_1 describes the free (Gaussian) theory for which all interactions are irrelevant and no scaling and universality is expected. The point FP_2 is a standard nontrivial percolation regime [3] with irrelevant advection. The last point FP_3 embodies a nontrivial regime for which both velocity and percolation interactions are relevant and

corresponds to a new universality class. Because for points FP_1 and FP_2 the velocity field is irrelevant, their boundaries are not affected by the value of the parameter α . On the other hand the stability region of FP_3 heavily depends on it. For incompressible case $\alpha = 0$ the boundaries between the regions can be computed exactly and can be read out from Fig. 1(a). From (3) one can expect that the parameter η formally drops out of the theory. However, RG analysis [14] shows that η affects the boundaries of fixed points. Note that in the blank space other regimes, e.g. rapid-change model or turbulent diffusion, can be realized.

With increasing value of parameter α situation becomes more involved. Numerical analysis shows that up to $\alpha = 6$ the region of FP_3 spreads to the origin, which can be seen on Figs. 1 for different values of α . Wherever it was possible, we have given the explicit form of boundary conditions. The main observation is that compressibility in cooperation with percolation interactions can lead to a stabilization of the nontrivial regime.

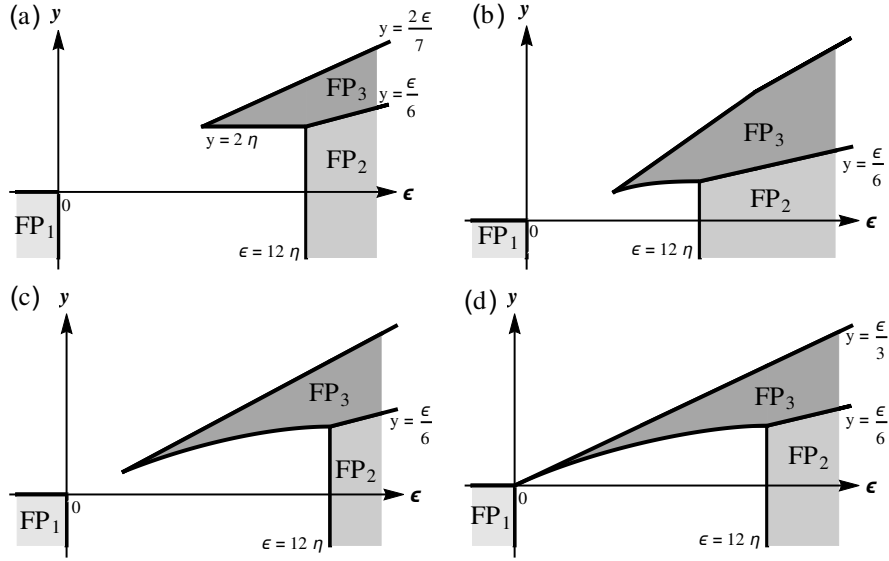


Figure 1: Regions of stability in the plane (ϵ, y) for different choices of the compressibility parameter α , which has been chosen to attain values 0(a), 1(b), 5(c) and 6(d).

For $\alpha \geq 6$ the lower boundary of the regime FP_3 can be computed exactly and is given by the following expression

$$y = \frac{6(\alpha - 3)\epsilon + [\alpha(2\alpha - 21) - 54]\eta - 3\sqrt{D}}{(\alpha - 3)(\alpha + 30)}, \quad (6)$$

where D is the quadratic polynomial in the expansion parameters η, ϵ

$$D = [1764 + \alpha(49\alpha - 372)]\eta^2 - 4(\alpha - 42)(\alpha - 3)\epsilon\eta - 12(\alpha - 3)\epsilon^2. \quad (7)$$

This situation is depicted on Fig. 2. With increasing value of α the regime FP_3 is stable also in the region with negative values of ϵ and the bottom boundary is given by Eq. (6). As FP_3 can be IR stable also for $\epsilon < 0$, which corresponds to the space dimensions $d > d_c = 4$, we conclude that velocity fluctuations can destroy the expected mean-field behavior. In the case of the pure irrotational (curl-free) field $\alpha \rightarrow \infty$ 2(b) the bottom boundary becomes a line $y = 2\eta$.

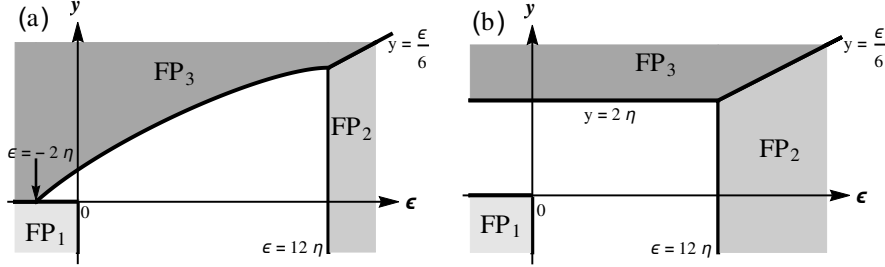


Figure 2: Phase diagram obtained for $\alpha = 12$ (a) and in the limit of pure irrotational velocity field $\alpha \rightarrow \infty$ (b).

From the theoretical point of view a special interest is paid to the three-dimensional Kolmogorov regime [17], which is obtained for the choice $y = 2\eta = 8/3$ and $\epsilon = 1$. From Fig. 2 it follows this regime lies precisely on the boundary of the regime FP_3 . According to detailed numerical analysis [14] this regime competes with another nontrivial regime and exhibits non-universal behavior, i.e. dependence on the initial value of bare parameters.

4 Conclusions

In this paper we have studied percolation spreading in the presence of frozen velocity field with long-range correlations. The field theoretic formulation of the model has been formulated and the consequences following from its multiplicative renormalizability have been discussed. We have found that depending on the values of a spatial dimension $d = 4 - \epsilon$, scaling exponents y and η , which describe statistics of velocity fluctuations, the model exhibits various universality classes. Some of them are already well-known: the Gaussian (free) fixed point, a directed percolation without advection and the last

one which corresponds to new universality classes, for which mutual interplay between advection and percolation is relevant. As has been shown [18] the main drawback of the model is that anomalous scaling behavior is destroyed when α and y are large enough. Therefore only relatively small values of α are admissible. In order to investigate the case of strong compressibility and clarify the role of compressibility one should try to apply more sophisticated model for the velocity fluctuations, e.g. one considered in [19].

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